Study on Structural Health Monitoring Method by Recursive Subspace Identification Based on Shaking Table Tests of Wooden Structure

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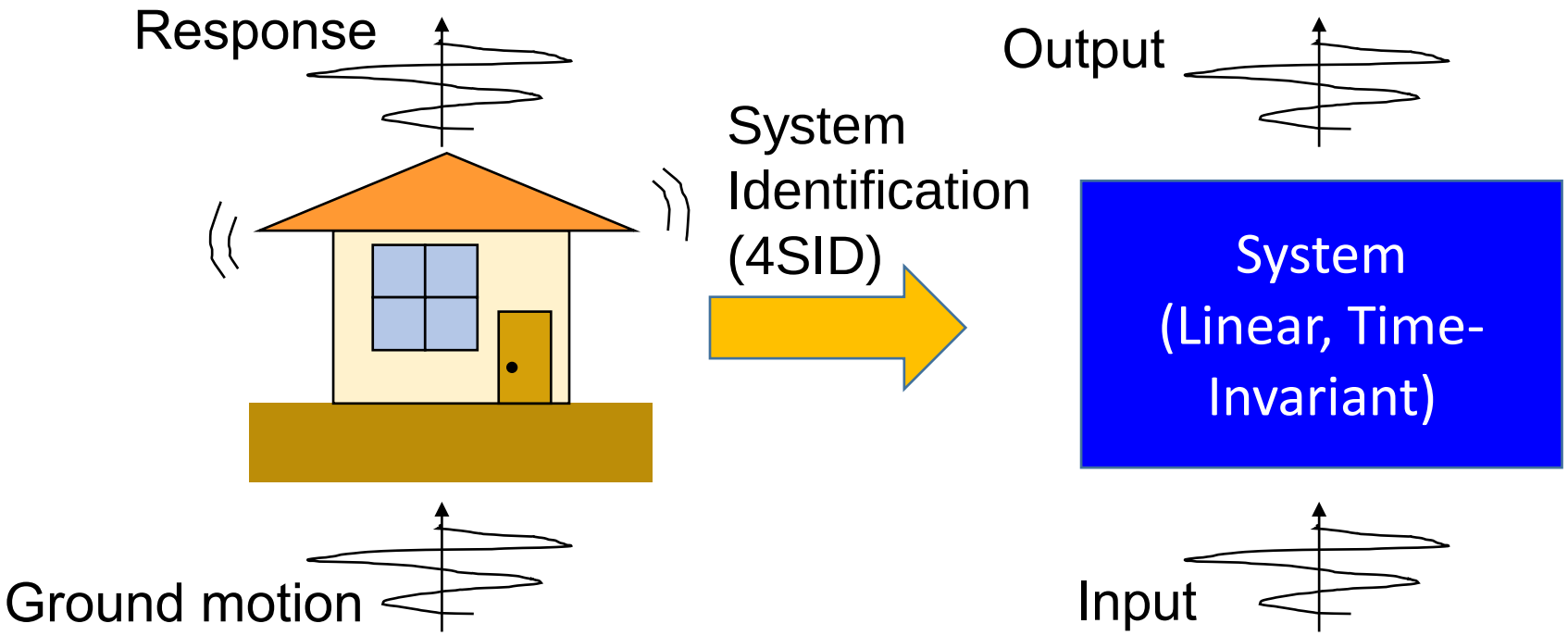
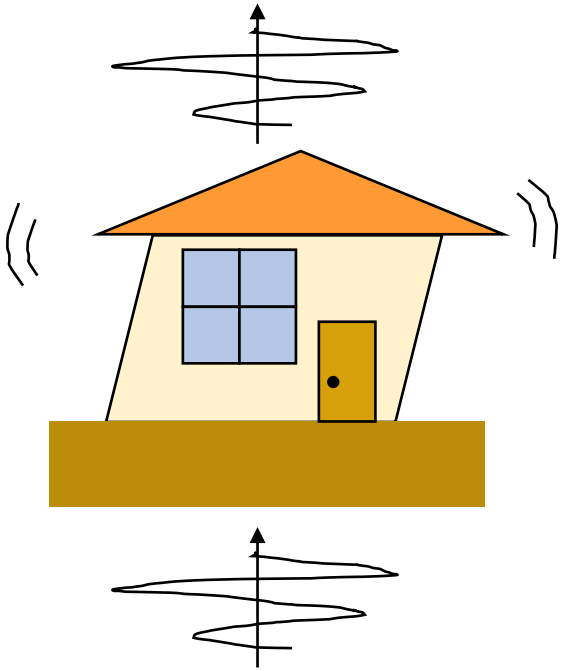
Introduction



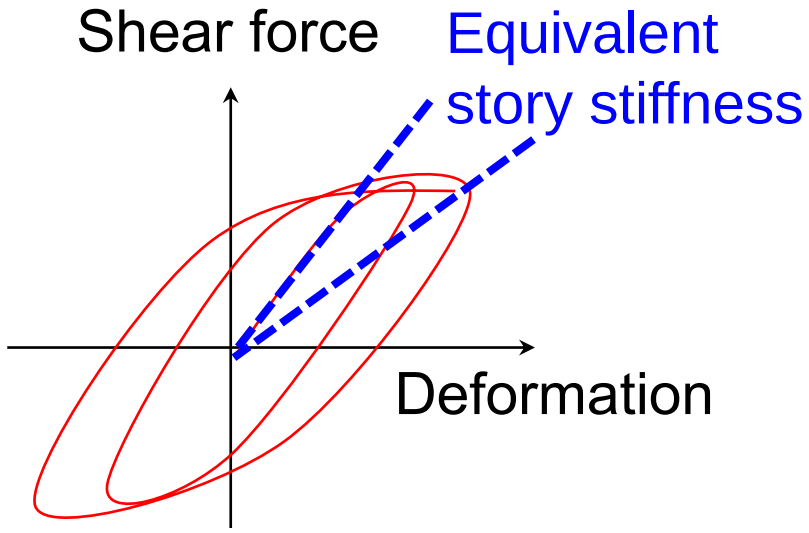
Concept of system identification

Subspace State Space System IDentification (4SID) :

- Used as one of the SHM methods.
- Time domain system identification method
- Evaluates the seismic response characteristics.
- Treats the building as a linear system.



Nonlinear behavior



If a building suffers damages during an earthquake,

- Building shows nonlinear behavior
- Seismic response characteristics of a building varies during earthquake (Time-variant system).

Recursive 4SID Method (Oku,1999):

- Applicable to time-variant systems.

Objective

- Response characteristics identified through non-recursive and recursive 4SID methods are compared based on Full-scaled shaking table tests of wooden structures.

- To clarify the advantages of recursive 4SID.

Methodology of non-recursive 4SID

State Space Representation

$$\mathbf{x}(N+1) = \mathbf{A}\mathbf{x}(N) + \mathbf{B}\mathbf{u}(N) + \mathbf{v}(N)$$

$$\mathbf{y}(N) = \mathbf{C}\mathbf{x}(N) + \mathbf{D}\mathbf{u}(N) + \mathbf{w}(N)$$

N: step
x: state vector
u: Input data vector (Input motion)
y: Output data vector (Response of structure)
v, w: Noise
A, B, C, D: Constant Matrix

Block Hankel matrix

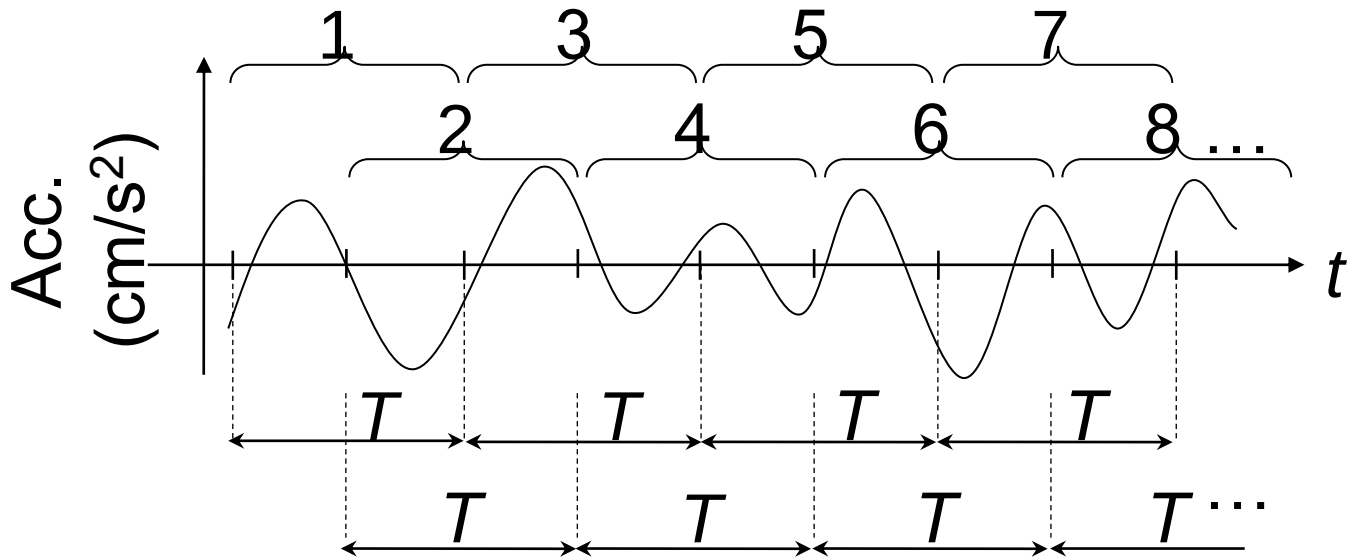
$$\mathbf{U}_N = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \cdots & \mathbf{u}_{N-\nu+1} \\ \mathbf{u}_2 & \mathbf{u}_3 & \cdots & \mathbf{u}_{N-\nu+2} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{u}_\nu & \mathbf{u}_{\nu+1} & \cdots & \mathbf{u}_N \end{bmatrix}$$

$$\mathbf{Y}_N = \begin{bmatrix} \mathbf{y}_1 & \mathbf{y}_2 & \cdots & \mathbf{y}_{N-\nu+1} \\ \mathbf{y}_2 & \mathbf{y}_3 & \cdots & \mathbf{y}_{N-\nu+2} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{y}_\nu & \mathbf{y}_{\nu+1} & \cdots & \mathbf{y}_N \end{bmatrix}$$

$\boldsymbol{\Xi}_N = \mathbf{Y}_N \boldsymbol{\Pi}^{\perp}_{\mathbf{U}_N}$ ⇒ Singular Value Decomposition

$\boldsymbol{\Pi}^{\perp}_{\mathbf{U}_N}$: geometric operator projecting the row space of a matrix onto the orthogonal complement of the row space of the matrix $\mathbf{U}_N$

Procedure of non-recursive 4SID



- Acceleration time history data are separated into several time segments.
- Response characteristics are then identified in each segment.

Methodology of recursive 4SID

State Space Representation

$$\mathbf{x}_{N+1} = \mathbf{A}_N \mathbf{x}_N + \mathbf{B}_N \mathbf{u}_\nu(N) + \mathbf{v}_\nu(N)$$

$$\mathbf{y}_\nu(N) = \mathbf{C}_N \mathbf{x}_N + \mathbf{D}_N \mathbf{u}_\nu(N) + \mathbf{w}_\nu(N)$$

x: state vector
u: Input data (Input motion)
y: Output data (Response of structure)
A_N, B_N, C_N, D_N: Time dependent

Block Hankel matrix

$$\mathbf{U}_N = \begin{bmatrix} \lambda \mathbf{U}_{N-1} & \mathbf{u}_\nu(N) \end{bmatrix}$$

$$\mathbf{Y}_N = \begin{bmatrix} \lambda \mathbf{Y}_{N-1} & \mathbf{y}_\nu(N) \end{bmatrix}$$

$$0 < \lambda < 1$$

$\gamma = \lambda^2$: Forgetting factor

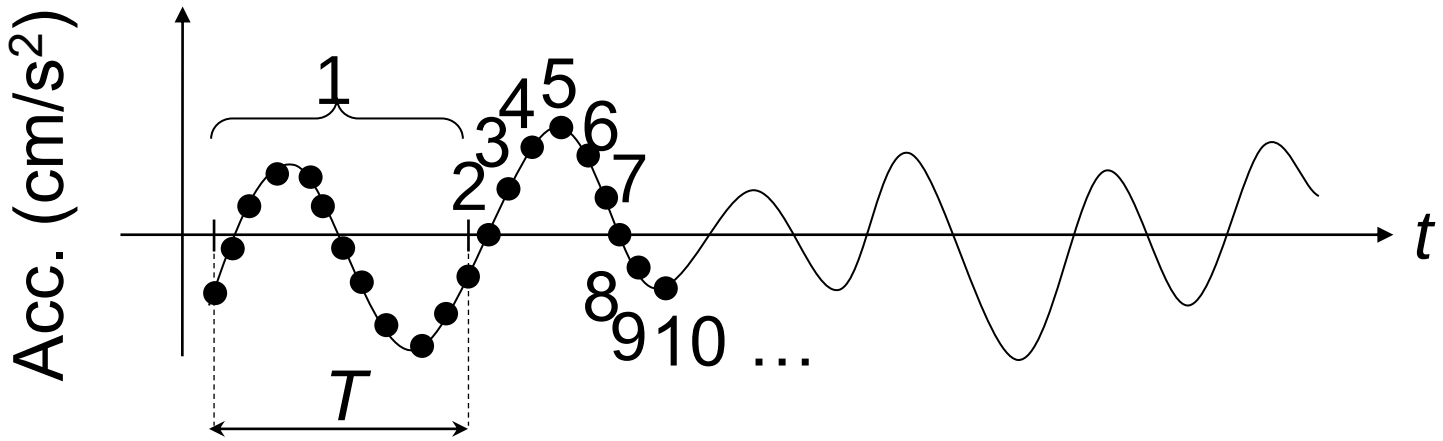
Compressed Input-Output Matrix

$$\mathbf{\Xi}_N = \mathbf{Y}_N \mathbf{\Pi}^{\perp}_{\mathbf{U}_N} \mathbf{Y}_N^T$$

$$\mathbf{\Xi}_N = \gamma \mathbf{\Xi}_{N-1} + \gamma \alpha_N \mathbf{e}_N \mathbf{e}_N^T$$ ⇒ Singular Value Decomposition

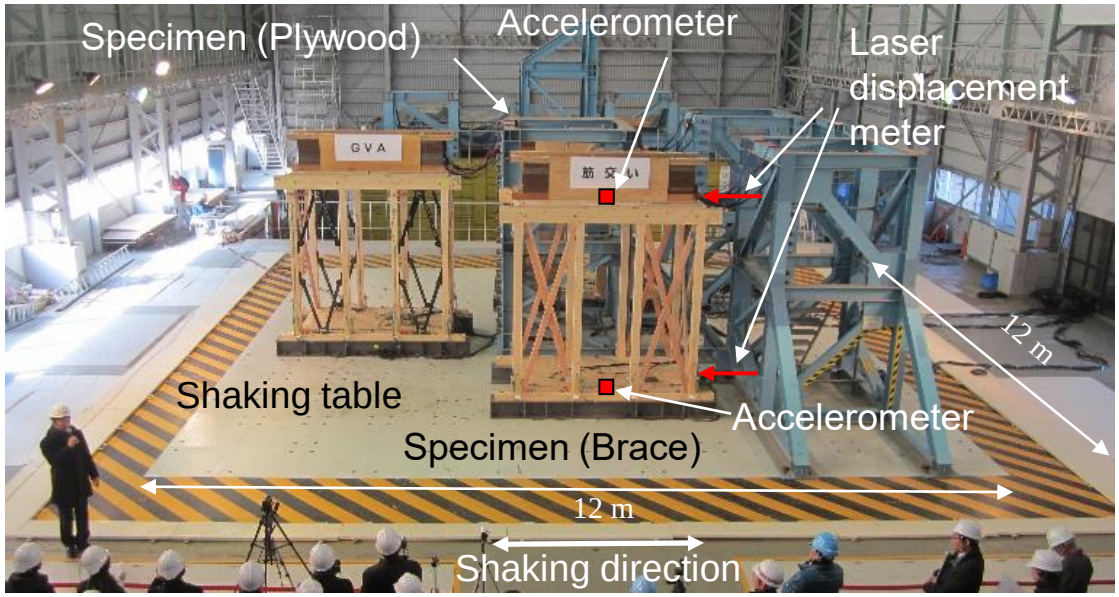
$\mathbf{\Xi}_N$ Could be evaluated every step, recursively.

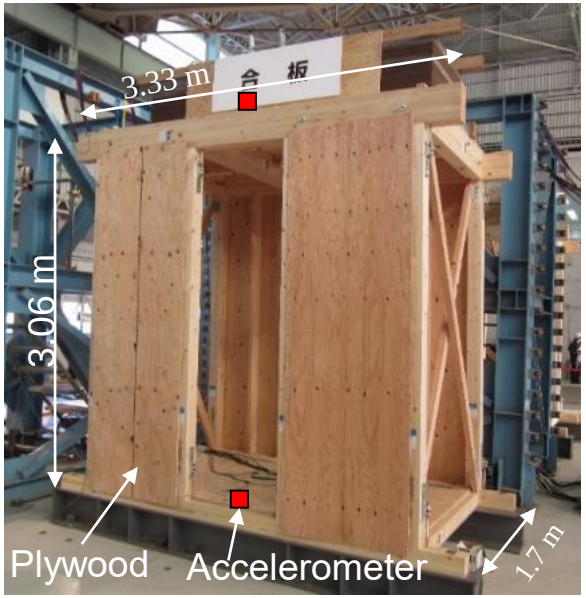
Procedure of recursive 4SID



- Data in time *T* were used to identify the initial response characteristics of the system.
- System identification is performed at each step, recursively.

Outline of Shaking Table Test

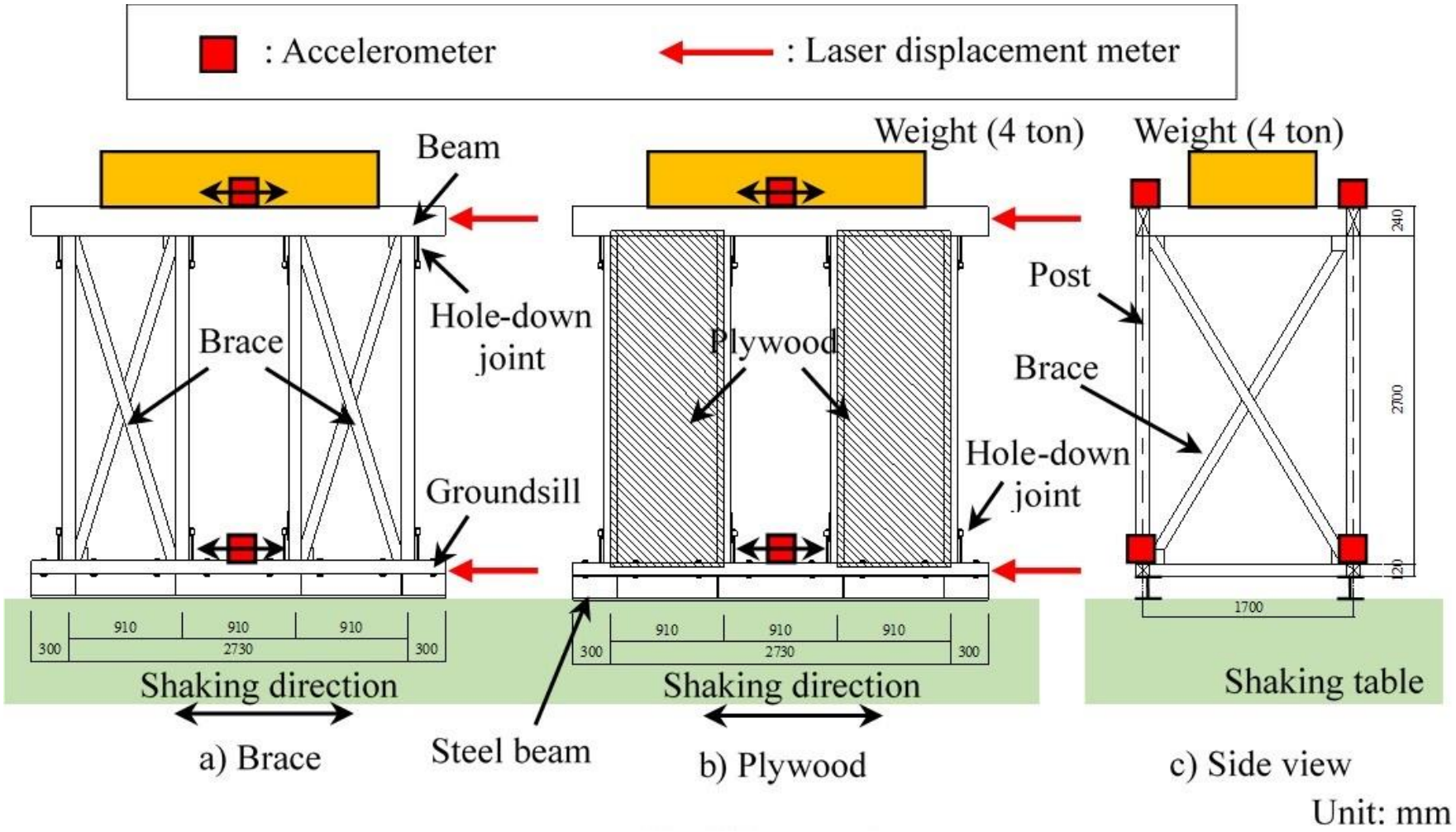




Specimen (Plywood)

* National Research Institute for Earth Science and Disaster Prevention in Japan

Test Specimens



Input Motion

Case 1	BCJ-Lv.1 (50%)
Case 2	BCJ-Lv.1 (100%)
Case 3	BCJ-Lv.2
Case 4	JMA Kobe

- Specimens suffered severe damages during JMA Kobe.
- In this study, only the JMA Kobe case is investigated.

Damages of specimens (After JMA Kobe)

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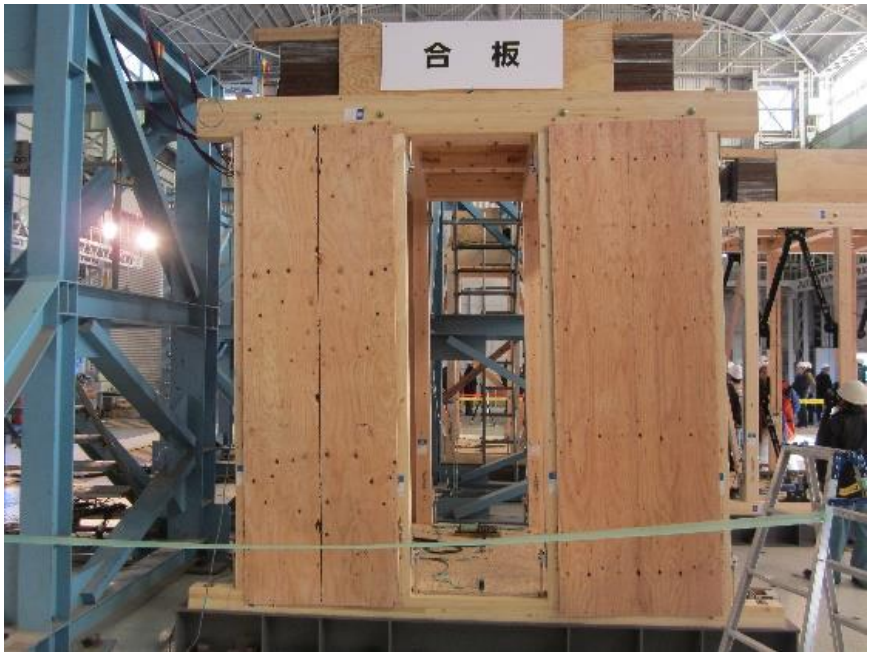
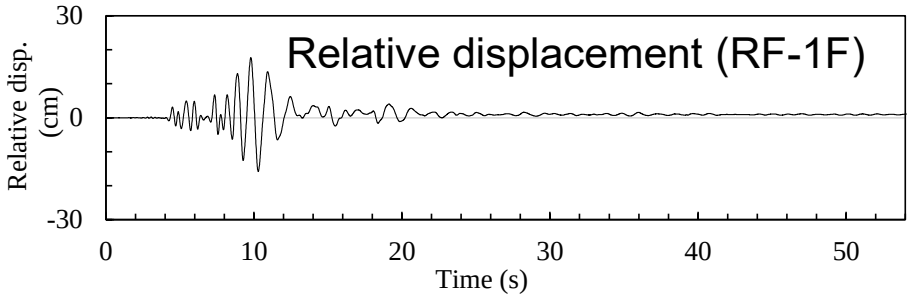
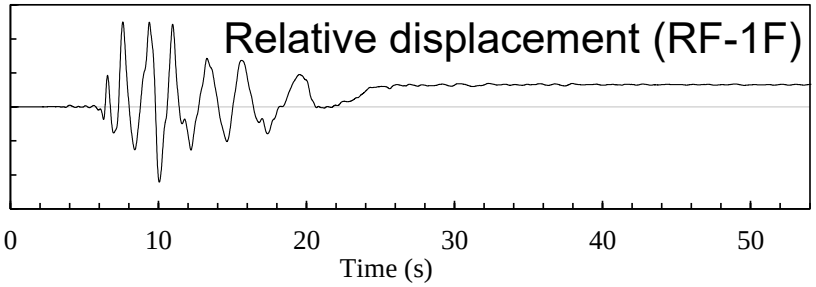
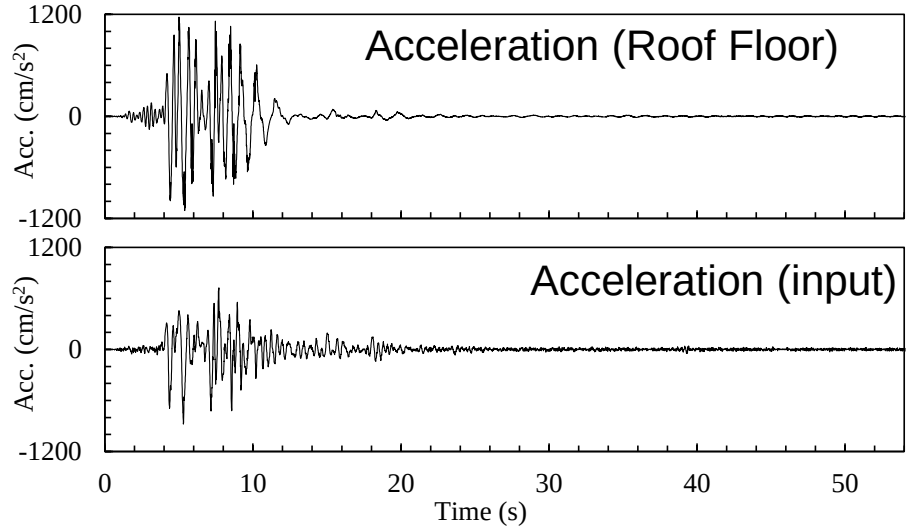


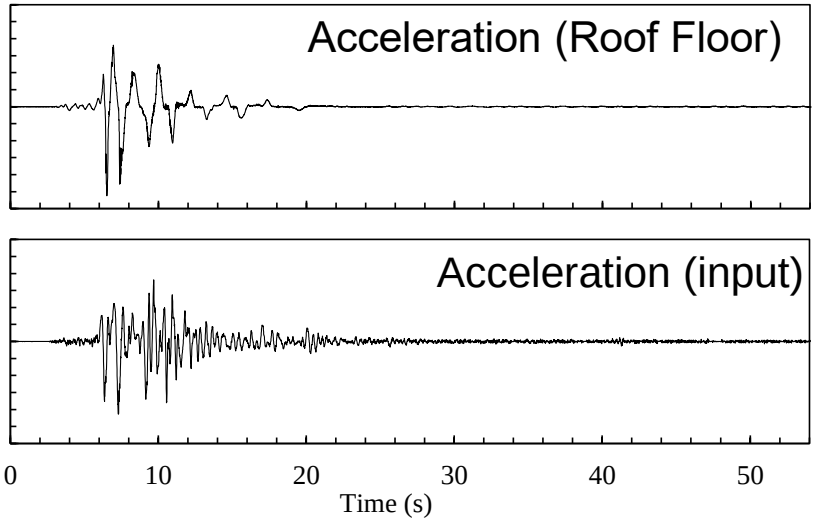
Plywood

Time History of Story Drift and Acceleration (JMA Kobe)





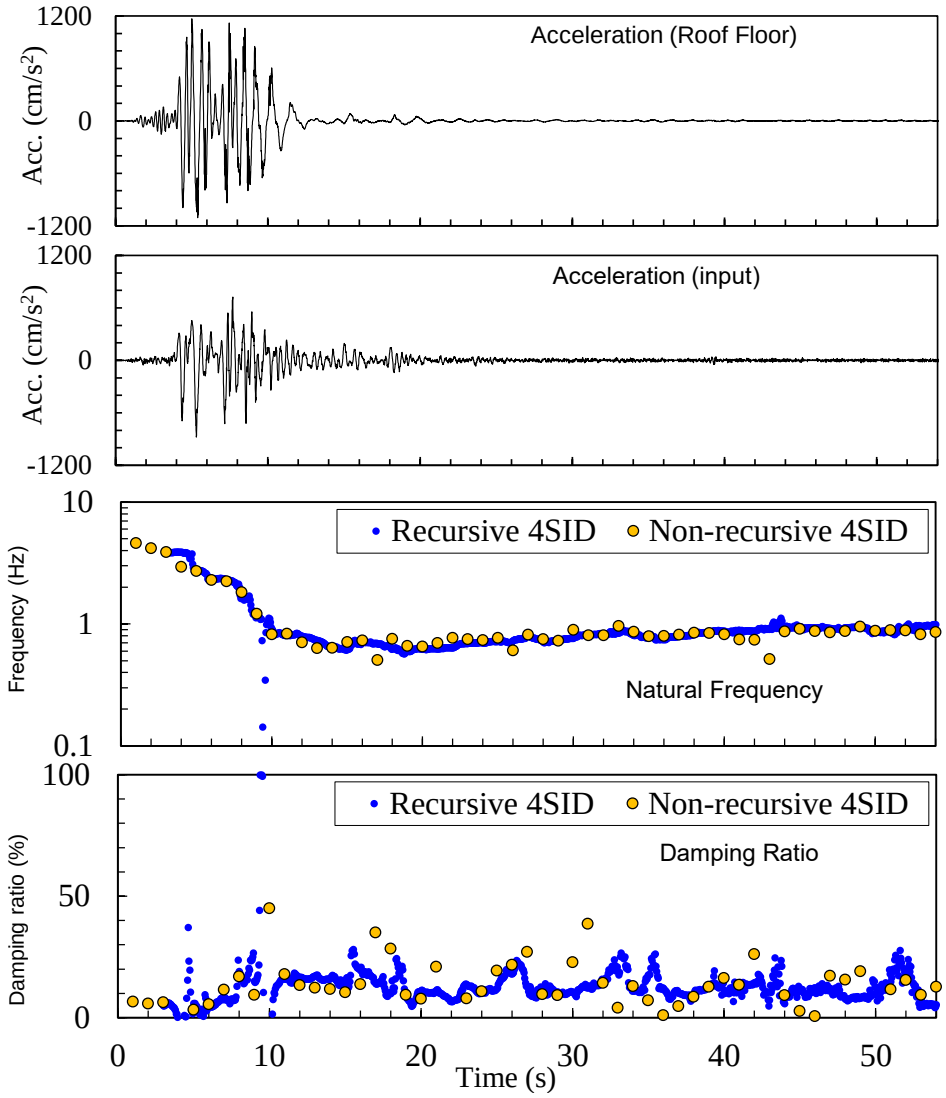




Brace

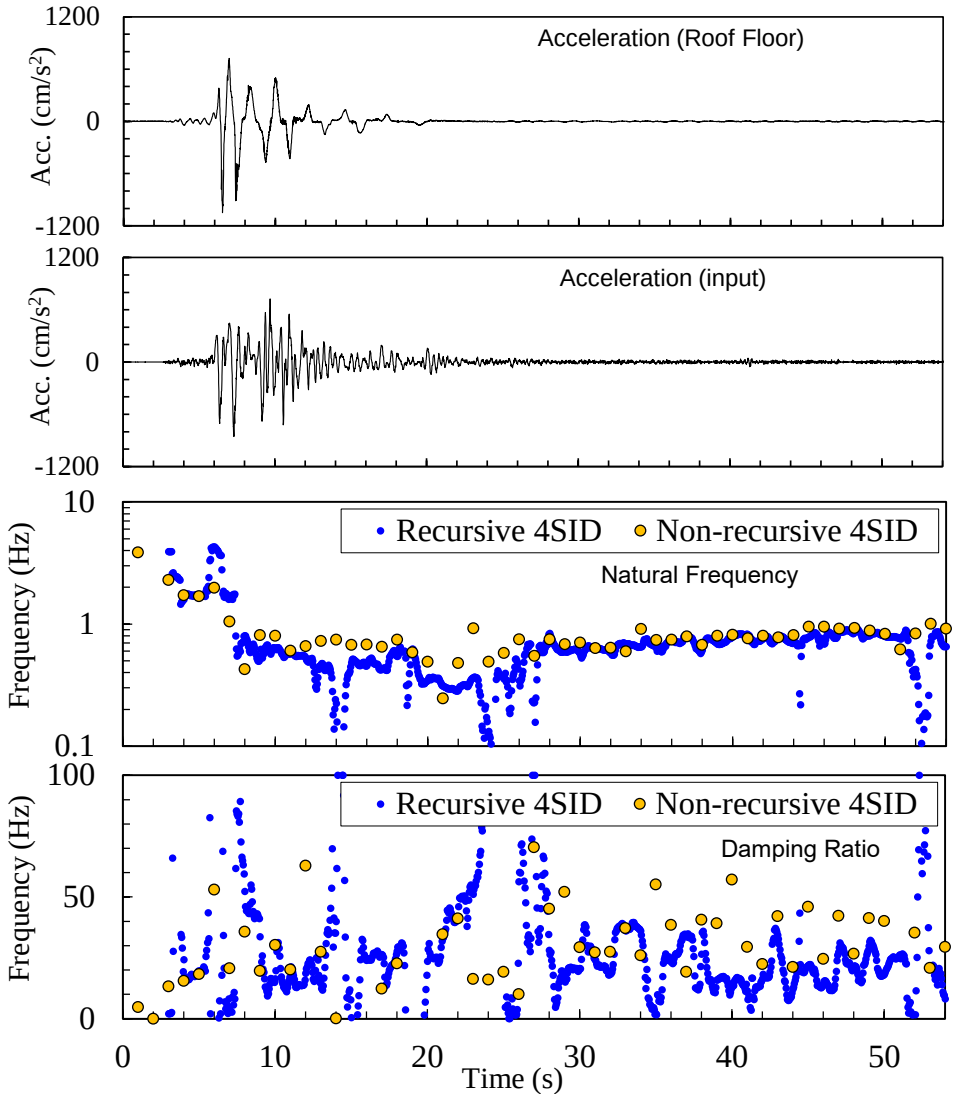
Plywood

Time-dependent response characteristics (Brace)



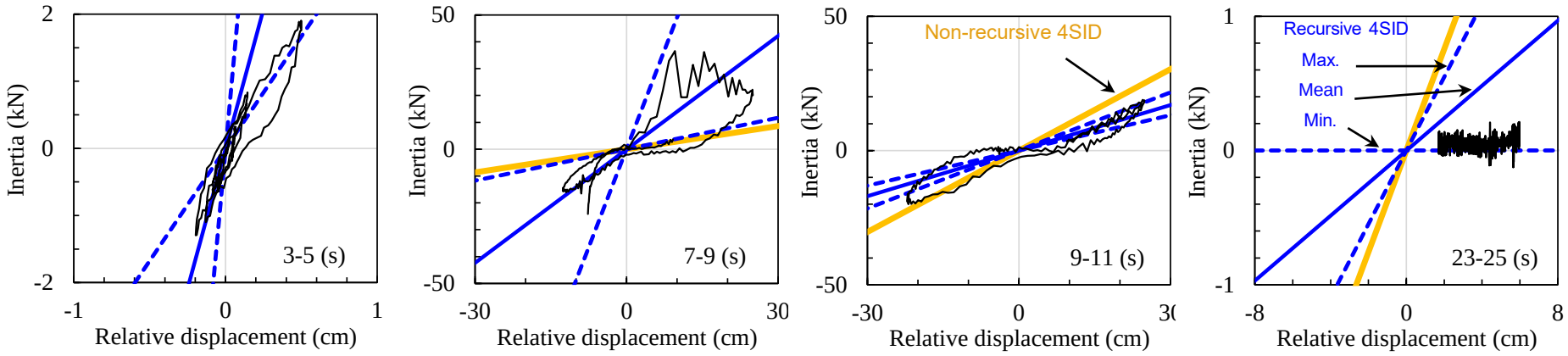
- Natural frequency degraded drastically with increasing acceleration amplitude.
- Natural frequencies and damping ratios identified using the recursive 4SID agreed with that identified by the non-recursive 4SID.

Time-dependent response characteristics (Plywood)



- Natural frequency identified by recursive 4SID approached to zero at 14 s, 24 s, and 52 s.
- Natural frequency identified by non-recursive 4SID did not show such behavior.

Verification of Identified Story Stiffness (Plywood)



- Structural damage might be underestimated if the non-recursive 4SID approach was applied to strong motion records of a structure showing strongly nonlinear behavior.
- Recursive 4SID approach is suitable for structural health monitoring when a structure shows a strong nonlinear behavior during an earthquake.

Conclusion

- If a building showed strongly non linear behavior, non-recursive 4SID method could underestimate the degradation of story stiffness.
- Instantaneous reduction of story stiffness can be evaluate by using recursive 4SID Method.
- Recursive 4SID method is suitable for SHM when a structure shows strongly nonlinear behavior during an earthquake.